Outcomes

- To maintain a positive tone for the class by promoting conversation and connections between mathematical concepts.
- To use paper strips as a manipulative for illustrating the connections between fractions and decimals.
- To use calculators as a tool to investigate patterns in the decimal representation of fractions.
- To introduce participants to the **NCTM Reasoning and Proof Standard**.

Overview

In the sixth session of Thinking About Fractions, Decimals and Percents, participants will use paper strips and calculators to examine the relationship between fractions and decimals. Paper strip activities will connect the fraction concepts learned in the first four sessions with the decimal concepts developed in session five. In this session calculator technology is utilized as a tool for investigating the decimal representation of fractions.

Time

15–20 minutes	The first part of the session allows participants to share their solutions to the homework problems.
50–60 minutes	Next participants make a set of fraction strips to explore the relationship between fractions and their equivalent decimals. Once this equivalency is established, long division is used to convert fractions to decimals. Technology enables participants to make explorations that used to be incredibly tedious. Using calculators, they explore fraction families.
10–15 minutes	This week's NCTM Standard is the Reasoning and Proof Standard. It is introduced midway through the lesson and reiterated in the final activity of the lesson.
10–20 minutes	In the closing activity parents reflect on the experience of making and testing mathematical conjectures.

Materials

| **Facilitator**	**Transparencies (English & Spanish)**
Fraction-Decimal-Percent Chart created in Session 1	*BLM 31: NCTM Reasoning and Proof Standards*
Participant	**Handouts (English & Spanish)**
- A calculator for each participant - A pair of scissors for each participant - One rubber band for each participant - One sheet of 12 x 18 construction paper for each participant 16 additional sheets of 12 x 18 construction paper - Clear tape for each table **Fraction Family Cards** copy on cardstock and cut into 16 cards per group of two or three - Extra copies of **Fraction Strips A and B**	**One per participant on cardstock** *BLM 32: Fraction Family Cards* **One per participant for class** *BLMs 33-34: Fraction Strips A and B* *BLM 35: Fraction-Decimal Summary Chart* **One per participant for home** *BLM 36: Bringing Mathematics Home 6* *BLM 37: Decimal Problems II*

Activities

Preparation of Classroom	Notes
1. Post the **Fraction-Decimal-Percent Chart** from the previous sessions. 2. Set up the **Chart It!** 3. Place the name cards from last class near the front of the room where participants can easily find them.	
Discussion of Homework (15-20 minutes)	
1. Tell participants that they will have a chance to share their internet experiences in the next class, since today's **Bringing Mathematics Home-6** offers more web sites to visit. Another reason for postponing the discussion until next time is to give participants who had difficulty logging on or finding access to computers additional time to do so. 2. Using **Decimal Problems I** - In small groups, have them share their explanations for correcting the errors on **Decimal Problems I**. - Ask groups to present their explanations to the class as if they were helping to correct errors made by their own children. - For each problem on the sheet ask for at least two explanations which use different models (number strips, base ten blocks, money) in the presentation. - Encourage discussion and interaction as groups present.	
Paper Strip and Calculator Activities (50-60 minutes)	
Connecting Fractions to Decimals with Paper Strips 1. Introduce the lesson by saying: *Take out the two Decimal Strips you made in the last class. One of these is labeled in fractions (tenths) and the other uses decimal notation.* *Line them up and place them next to each other and observe that 6/10 = 0.6. This means that they are equivalent.* *In the next activity, we will find decimal equivalents (or approximations) for some other fractions.* 2. Hand out **Fraction Strips A and B** and a pair of scissors to each participant.	

Activities

Paper Strip and Calculator Activities (continued)	
3. Say: • *Notice that below each strip is a fraction "family" name.* • *Cut out the strip for FOURTHS and notice that the fourths are marked for you.* • *Your job is to cut, mark, and label the other strips according to the fraction family. Without using a ruler to measure, mark the strip. You may fold it or compare it to the marks on another strip.* • *Marks need not be perfect, but should be reasonably accurate.*	The intention of this activity is to give participants the opportunity to employ problem-solving skills while applying their fraction understanding. For this reason, it is best not to provide specific directions for how to determine the lines marking each strip.
4. Ask participants to discuss their strategies for making the strips. Encourage the sharing of alternative approaches to the task. 5. Hand out one sheet of 12" x 18" construction paper to each participant and one roll of tape at each table.	If some participants work slowly or have difficulty, they can be encouraged to collaborate with another person to make one set of Fraction Strips to share.
6. Instruct them to tape the fraction strips in order (halves, thirds, fourths, fifths, eighths, tenths, twentieths) on the sheet so that the bottom (zero line) of each strip is in line. (Do not include the decimal strip yet.) 7. Ask participants • *What fraction equivalents do you see by examining the array of strips?* • *How might these strips be used to compare fractions?* 8. Now pose this question: *How could these strips be used to find the decimal equivalents (or approximations) for fractions?* 9. Illustrate how to use the fraction and decimal strips to make these comparisons with commonly known decimal equivalents (Example: 1/2 = 0.5 and 1/4 = 0.25). • Say: *Find decimal equivalents for other fractions by using the strips in this way: One partner names a fraction on one of the strips. The other partner finds the decimal equivalent (or approximation) for that fraction. Trade jobs and repeat the process.* • Circulate and listen as participants work to make sure all are successful. • Ask: *While using the strips is a reasonable way to find decimal approximations for fractions, it is not a completely precise method. Why is this method of finding the decimal equivalents of fractions not completely reliable?*	Participants should tape strips close to one another or even touching. In addition to the seven fraction strips, a decimal strip will be added in this session and a percent strip in session 8. If participants have difficulty seeing equivalents, suggest they take a rubber band that has been cut and stretch it across the array of strips to see which marks line up. Participants should suggest placing the previously made decimal strip in the array (lined up with the others) and checking to see where a particular fraction mark lines up with the decimal strip.

Activities

Paper Strip and Calculator Activities (continued)	Notes

10. Tell participants to save the paper with the strips taped to it and bring it back to session 8.
 	
Converting fractions to decimals using division
1. Explain that a more precise method for finding decimal equivalents for fractions is division. Remind participants that in session 3, the Cookie Sharing activity pointed to the connection between division and fractions. Use the examples of 1/2 = 0.5, 1/4 = 0.25, and 3/4 = 0.75 to demonstrate the long division process that converts a fraction to a decimal.
 	
2. Have participants use division to find the decimal equivalent for 3/20 and verify the equivalence using the number strips.
 	
3. Have participants use division to find the decimal equivalent for 2/3.
   • Participants will see that the division does not end, but produces a repeating pattern.
   • Tell them that decimals such as this are called repeating decimals and that decimals that come out even (like the previous examples) are called terminating decimals. Chart It!
   • Ask them to return to the decimal strips and see that 0.666666… appears to be a reasonable equivalence for 2/3.
 	
4. Have them do one more long division example: 2/11. This produces a two-digit repeating pattern, 0.1818181.
 	
5. Say:
   • *Long division is accurate, but is a time consuming and tedious process especially if a decimal has many digits in its representation.*
   • *Recall that NCTM's Technology Principal introduced in session 5 states that technology is essential and enhances student learning. A calculator can be used to perform otherwise tedious calculations like turning fractions into decimals.*
 	
6. Hand out a calculator to each participant.
   • Demonstrate how to use it to perform the same division problems they have just completed.
	Participants will likely point out that the strips were made in a fairly imprecise way (folding) and the decimal strip only has precision to tenths and the hundredths place.
	
	Be sensitive to the fact that some participants may not be comfortable performing long division. Encourage partners to work together.
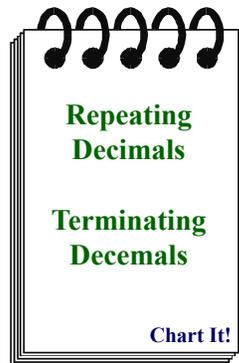	
	Repeating Decimals
	
	Terminating Decemals
	
	Chart It!
	
	At this point, you may introduce the repeating bar notation as a shorthand way to represent repeating decimals. 2/11 = 0.18

Activities

Paper Strip and Calculator Activities (continued)	Notes
• Point out that some calculators round the last digit, and for this reason the final digit may be disregarded when looking for repeating patterns. • Ask them to divide using the calculator to find the decimal for 5/6. • Point out that in this case, the repeating pattern starts in the hundredths. It would look like this: 0.8.3 when written using the repeating bar. **Fraction Families** The next activity is an exploration of fraction families. The goal is to convert fraction families into decimals. Participants look for patterns, generalizations, and relationships within and between families. 1. Model the activity. Tell participants that the group assigned the **sixths family**, would do the following: • Write all the fractions in the sixth family between 0 and 1: 1/6, 2/6, 3/6, 4/6, 5/6 • Cross out all "imposters." Imposters are fractions that are not in lowest terms. 1/6, 2/6, 3/6, 4/6, 5/6 • Use the calculator to change "true" family members into decimals. • Write these in large print using a marker on a sheet of 12" x 18" paper. These will be posted in the room and analyzed later in this class session. 2. Have groups work on **Fraction Family Cards**. • Hand out one **Fraction Family Card**s to each group of two or three participants. • Groups that finish early can be given a second family to work on. • Let participants know that some groups have been assigned "small families" (families with few true family members) while others have been assigned "large families" (families with lots of true family members). • Their posters do not need to include all family members; simply record as many as will fit (maybe 5 or 6 members). • As they complete their posters, display them in the room in consecutive order. You will return to these posters after presenting this week's NCTM Standard.	Make a poster written with large print to serve as an example. See example below. **Sixths** $1/6 = 0.1\overline{6}$ $5/6 = 0.8\overline{3}$ Remind participants to write large enough for these to be read from across the room. Families for sevenths, thirteenths, and fourteenths have not been included on **Fraction Family Cards** because of their lengthy decimal representation. A complete list of fraction families is included on page 10. If desired, you can make the posters for these omitted families so that they can be included in the display.

Activities

Connections to National Standards (5 minutes)	Notes
1. Display the **NCTM Reasoning and Proof Standard** transparency. Say: *One of NCTM's process standards is the Reasoning and Proof Standard. This standard includes these ideas:* *Instructional programs from pre-kindergarten through grade 12 should enable all students to—* • *Make and investigate mathematical conjectures.* • *Develop and evaluate mathematical arguments and proofs.* 2. Tell participants that the next portion of the lesson will give them the chance to experience this standard as they analyze and investigate the Fraction Family posters they made.	
Fraction Family Card Activity (10-15 minutes)	
1. Direct participants to look at the set of posters made in the last activity. 2. Ask them to see what kind of patterns, generalizations,or observations they can make about the decimal representations of fractions. 3. Encourage participants to provide examples to support their observations. 4. Encourage generalizations and discuss how these generalizations could be tested. 5. Questions to promote discussion: • *What traits did you notice within your family?* • *What traits seem similar between families?* • *Which families might be "relatives" of one another?* • *What do all terminating (or all repeating) families have in common?* • *How could we predict if a new family would repeat or terminate?*	If desired hand out copies of **Fraction-Decimal Summary Chart** so that participants have a personal copy of this data to take home. Participants might notice: • in a particular family either all members repeat or all members terminate using the same number of digits • similarities between thirds, sixths, ninths, twelfths • similarities between halves, fourths, eighths, sixteenths • similarities between other families whose denominators share factors The last question is more difficult. It is not important for participants to come to a conclusion, but for them to make and support conjectures.
Closure (15-20 minutes)	
Participants reflect on the experience of making and testing conjectures in this math class. Ask: • *How is this like/unlike the experiences you remember from previous math classes?* • *What value do you see in making and testing conjectures?* • *How does it make you feel?*	Be prepared for some participants to express a lack of comfort with making and testing conjectures. Allow them to express this and probe to find out why they feel this way.

Activities

Take Home Activities (5 minutes)	
1. There are two items for participants to take home: **Bringing Mathematics Home 6** and **Decimal Problems II**. 2. Discuss options for getting on the internet for those who do not have prior experience with this technology. Participants might visit the public library or their child's school to find computers with internet access. 3. The directions for completing **Decimal Problems II** may be confusing to some participants. It will help if you demonstrate the directions. • They are to move through the maze from **0** to **1** but must always move from a smaller value to a larger value. • It will be useful to turn the fractions to decimal in order to make size comparisons. Let them know that there is more than one solution path.	Parents will need access to computers to complete the **Bringing Mathematics Home 6** activities. Some parents will not have access to the internet. If computers are available in the Math For Parents classroom, it would be a great idea to invite parents to come early or stay late to the next few sessions and look at the web pages together. Another option is to arrange to meet several of them at a library during the week and log on together!
Preparation for the Next Session (5 minutes)	
1. Collect name cards for use in the next sessions. 2. Fold or roll the **Fraction-Decimal-Percent Charts** in a way that preserves the items posted on them and bring them to the next class. 3. Save the **Chart It**! and bring it to the next class. If desired, you may have the log typed and distributed to participants at the next class.	