SESSION ONE
ADDITION OF WHOLE NUMBERS

Outcomes

- To use base ten blocks to represent and add numbers
- To use a counting board and beans to represent and add numbers
- To understand how adding numbers that involve carrying is similar to using paper and pencil when using manipulatives
- To do investigations involving addition

Overview

The first session of Thinking About Numbers focuses on understanding how methods of addition work. The activities involve representing numbers and adding numbers by using base ten blocks, counting board and beans, and paper and pencil. By using the manipulatives, participants will gain a clearer picture as to how the carrying process works when using paper and pencil for addition. The session ends with some addition investigations.

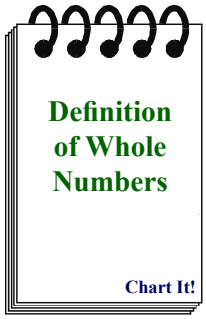
Time

10–15 minutes	The first part of the session allows participants to introduce themselves to one another and talk about their children.
30–40 minutes	Next participants work with a partner to represent and add numbers using base ten blocks. When the participants do the same problems using paper and pencil, they are given a chance to make connections between the two methods.
30–40 minutes	The next activity is a little more abstract then the base ten blocks. It involves representing and adding numbers using beans and a counting board. With this activity participants get hands on experience of trading groups of tens. Again participants have an opportunity to connect this with the process used with paper and pencil.
15–20 minutes	After the explorations with place value, participants do some investigations involving addition. These are more abstract then the other activities. They involve being able to make sense of conjectures and testing them for validity.
15–20 minutes	In the closing activity, parents think about how they learned mathematics and how it differs from the way that mathematics was taught in this first session.

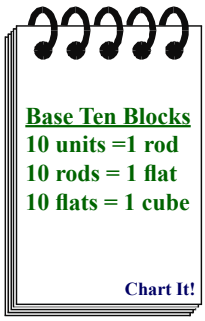
Materials

Facilitator	Transparencies (Eng. & Spanish)
• A set of base ten blocks for the overhead • Transparent colored chips	*BLM 1: Counting Board*
Participant	**Handouts (English & Spanish)**
• A set of base ten blocks: 20 units, 30 rods, 15 flats, and 3 cubes for each two participants • Beans • Straws (at least 2 per participant)	**One per participant for class** *BLM 1: Counting Board (card stock)* **Two per participant for class and home** *BLM 2: Addition with Base Ten Blocks* *BLM 3: Addition with a Counting Board* *BLM 4: Addition Principles* *BLMs 5-7: Base Ten Templates (card stock)* **One per participant for home** *BLM 8: Bringing Mathematics Home 1* *BLM 9: Addition with Whole Numbers*

Activities

Preparation of Classroom	Notes
Since there are several take home activities in this session, it would help to make a packet for participants ahead of time. Have a supply of base ten blocks on the tables.	Base ten blocks are also known as place value blocks and Diene's blocks.
Getting to Know Each Other (10-15 minutes)	
1. Introduce yourself and tell a little bit about your professional and personal background. If you have children be sure to talk about them. Parents will feel comfortable relating to you as a fellow parent. 2. Tell participants that this course is about number sense and is designed to serve three purposes: • To expand their understanding of mathematics • To experience mathematical activities that can be used with their children at home • To have fun! 3. Tell participants that the course will be more enjoyable if we get to know one another and learn each other's names. • Give each participant an index card. • Ask each participant to fold the card to form a tent. • Ask them to write their name in large letters and to write the first name and the grade level of each of their children on the other side of the card. 4. After the name cards are completed, ask participants to introduce themselves and share how many children they have in school.	
Addition Using Base Ten Blocks (30-40 minutes)	
Representing Numbers with Base Ten Blocks 1. Say: *This session is about addition of whole numbers. I know that you all know how to add. What this session attempts to do is to help you teach your children to understand addition. While addition may seem easy to you now, I'll bet when you were first learning how to add it was strange and difficult to keep your columns straight and carry your numbers correctly. This session attempts to explain why our methods of addition work the way they do. During this session we will be working with whole numbers. Whole numbers are numbers that you count with: 0, 1, 2, 3 etc.*	

Activities

Addition Using Base Ten Blocks (continued)	Notes
2. Write the following problem on the overhead and have participants work the problem: **5897** **+6319** • *Imagine that you are with someone who has never done an addition problem before and you are explaining how to add. Think about how you would justify the way you did the problem. Could you explain it to the class?* • *Our first four activities will involve adding with base ten blocks so that we can explore why we add the way that we do.*	
3. Ask participants to build 436 with their base ten blocks. Walk around the room to see how the participants do with this task.	It is nice to give participants an opportunity to explore with the blocks by building 436 before explaining the value of each piece of the set.
4. Demonstrate on the overhead and say: • *There are four types of base ten blocks: There are units, rods, flats, and cubes.* • *How many units make a rod?* After getting a response, model this by forming a rod out of 10 units. • *How many rods make a flat?* After getting a response, model this by forming a flat out of 10 rods. • *So, what is the value of a flat?* (Chart it!) • *If we have 5 rods and 3 units. What number do we have?* (Demonstrate this) • *What if we had 3 flats, 7 rods and 4 units?* (Demonstrate this) • Hold up the cube. Ask participants how many units would be in this cube if it were cut along all the lines. Give them a few moments to visualize that they would have 1000 units. After getting a response, model this by forming a cube with 10 flats. • *If we have 2 cubes, 3 flats, 5 rods and 3 units. What number do we have?* (Demonstrate this)	 Some refer to these lengths as: Unit Cubes (1's) Rods (10's) - also called "sticks" Flats (100's) Cubes (1000's)
5. Ask participants to represent three hundred fifty four (after a moment, demonstrate this). Have participants put these aside. Ask: *How can you represent two hundred eighty seven with base ten blocks?*	

Activities

Addition Using Base Ten Blocks (continued)	Notes
Give participants time to do this task, and then demonstrate it on the overhead projector. **Addition with Base Ten Blocks** 1. Now the participants will be adding the piles. • Say: *Now, see what happens when you add three hundred fifty four blocks plus two hundred eighty seven blocks by combining the two piles.* • Combine the two piles on the overhead, but do not do the problem yet. • Circulate to find out how the participants are doing. • Pick one that is capable of explaining what they did. Since this is the first class, and participants may be shy, have the participant explain to you how they added these blocks and what trades they did. Demonstrate the trades with overhead base ten blocks. • Point out to them that putting together the flats, the sticks and the blocks we get six flats, 4 sticks and one block. • Ask: *What number does this represent?* 2. Hand out **Addition with Base Ten Blocks** and have the participants work on this as you circulate the room and answer questions. 3. Have participants demonstrate their thinking with activities 1 and 2 on the overhead. 4. The processing of activity 3 is very important. • When adding the ones column: 6 and 9, the participants think 15, and think I write the 5 and carry the 1. They are actually getting 15 blocks, but trade them for one stick and 5 left over blocks. This is the reason that the 1 is carried. It is carried into the tens place, or stick place and we have one more stick than before. So the one that is carried represents one stick. • Make similar connections for the sticks that are exchanged for a flat.	Having participants share their thinking through presentations is a very important process. It will take extra time and will feel uncomfortably slow to participants the first few times. However, they soon will realize the benefits from trying to explain their thinking, as well as hearing other perspectives on the same problem. During this first session, it is likely that the participants will be shy about coming to the overhead to demonstrate their thinking. Help them gain confidence in their presenting by having them talk through their thinking from their table for the class while you demonstrate their thinking with the overhead manipulatives. Another alternative is to have them stand beside you at the overhead while you support them in sharing their thinking. This is reassuring to them because they know that you will help them if they get stuck. It is important to take time to process the paper and pencil connection to the manipulatives. The participants need to make clear connections between the one that is carried and the fact that it is one stick or one flat that is being added to the next grouping.

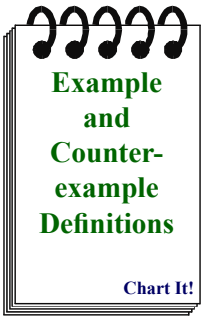
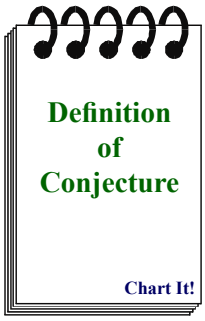
Activities

Addition With Beans and Counting Board (30-40 minutes)	Notes
Representing Numbers with Beans and a Counting Board 1. Display the **Counting Board** transparency on the overhead projector and pass out the handout with beans. Say: *We have been looking at representation of numbers with base ten blocks. There are many ways to explore numbers. The next activity will use beans and a counting board to represent numbers.* Write 4,381 at the top of the transparency and ask a volunteer to come up and show how that might be represented on the counting board. 2. Ask the participants to represent fourteen thousand three hundred eighty two. Ask: *In what order did you place the beans? So, not everyone put the beans on the counting board in the same order. Why is it that order did not matter here?* **Adding with Beans and a Counting Board** 1. Pass out **Addition with a Counting Board** and **Counting Board**. • As participants look at problem 2, have them represent one thousand eight hundred twenty nine. • Have them put a straw under that number to separate it from the next. • Have participants represent four hundred eighty three below the straw. • Have them remove the straw and add the two numbers, recording it on their paper. 2. Have a participant show their thinking about the problem on the overhead. 3. Ask if anyone added from left to right. • Have them demonstrate this on the overhead. • If no one added this way, demonstrate it. • Ask which method gives a quicker estimation of the sum. It is important for children to understand that they can estimate the sum by looking at the two highest place values. 4. Have participants do problem 3 with paper and pencil.	What you are looking for "In what order did you place the beans?", is the fact that when one knows place value, they can be flexible about the order of filling in the columns on the counting board. Parents might work from the right or the left on this problem or use other alternative methods. In today's classroom, children are encouraged to approach addition using various methods. Encourage and support the participants when they present alternative methods.

Activities

Addition With Beans (continued)	Notes
5. Have them think about how the paper and pencil process relates to the bean counting (see problem 4). 6. Illustrate the idea that adding could be done in columns with the trades showing: 1, 8 2 9 + 4 8 3 1 | 12 | 10 | 12 7. Ask participants to use the counting boards to work the problem 5 and the challenge problem on their own. 8. Have participants share how they solved each of the 2 problems. 9. Discussion question: • *Which methods have we used to do these addition problems?* • *Which method is easier for you? Why?* Let them know that blocks and beans are quite different and that the concepts should be taught first with blocks because the exchanges can be counted. Beans are more sophisticated and a higher level of thinking because you have the place values marked, but you do not have a visual of those values. The beans more closely parallel our paper and pencil method.	
Mathematical Reasoning (15-20 minutes)	
Tell participants that each session will include an exercise in mathematical reasoning connected to the subject of the session. This format is used on many standardized tests for students today, and will give you an idea of what is expected. Since the focus of this session has been addition, the mathematical reasoning will be about addition. 1. Hand out **Addition Principles**. Have them look at problem one and ask if the following statement is true or false: $A + B = B + A$ 2. Have participants vote on whether the statement is true or false. Then ask how many are just simply confused about what the statement could mean.	

Activities

Mathematical Reasoning (continued)	Notes
3. Explain the meaning of the statement by using an example like 3 + 5. • Write 3 + 5 and ask what it equals. • Then write 5 + 3 and ask what it equals. • Equate the 3 to A and the 5 to B and illustrate how this is an example of A + B and B + A and that they both equal the same amount. Therefore you can state that A + B = B + A. • Have them try some problems at their tables to see if they can find any example where this statement is not true. • The class should come to the conclusion that they cannot find an example where it is not true. • Let them know that if they had found one, it would be called a counter-example. 4. Have the participants explore the second statement, problem number 2 on the handout. • Ask them to look for examples and counter-examples of the statement. • After some minutes, have a volunteer show an example that fits this statement. • Have another volunteer show a counter-example. • Explain that even though some examples can be found, if one counter-example is identified, the statement is considered false. Therefore, this second statement is considered false. 5. Tell participants that many textbooks use the word conjecture to describe the kinds of statements that we will be exploring. Conjectures are suspected rules that can be examined. It is examined through examples and counter-examples. When working with conjectures, the object is to find a counter-example.	 
Closure (10 minutes)	
Participants reflect on the activities of this session and their experience with addition. • Direct them to share with a partner one mathematical idea they learned or saw differently as a result of this session. • Circulate and listen to this sharing. • Ask a few volunteers to share their reflections. • Record significant insights.	

Activities

Take Home Activities (5 minutes)	Notes
1. There are eight handouts for participants to take home: • **Bringing Mathematics Home 1** • **Addition with Whole Numbers** • **Addition with Base Ten Blocks** • **Addition with a Counting Board** • **Addition Principles** • **3 Base Ten Templates (copied on card stock)** 2. Have participants look through the packet of materials as you explain them. The object of the take home activities is for them to practice with their children. They have templates for making base ten manipulatives and fresh copies of the session's activities. 3. There are also some new challenges for the participants in **Addition with Whole Numbers**. 4. Let participants know that they should be ready to share their experiences at the next session.	
Preparation for the Next Session (5 minutes)	
1. Collect name cards for use in the next sessions. 2. Save the **Chart It!** and bring it to the next class. If desired, you may have the log typed and distributed to participants at the next class.	