

## SESSION FOUR

### TABLES AND MEANINGFUL EXPRESSIONS

#### Outcomes

- To complete a table and determine the appropriate expression for the table
- To write meaningful algebraic expressions

#### Overview

This session continues the exploration of expressions. Participants practice writing expressions from tables and create tables to reflect expressions that they have invented. They also examine expressions that are created from a list of variables representing everyday items. They determine which expressions make sense and then create several of their own. The NCTM Algebra Standard is introduced.

#### Time

- 10–25 minutes The session begins with a discussion about the *Money Exchange Game*, the patterns it generates and the methods for recording the moves.
- 25–30 minutes Participants work individually to complete tables. They decide on an expression for each table and compare their answers with others. They then make up an algebraic expression and complete a table for that expression. Each group challenges the entire class to find the expression for one of their tables.
- 30–40 minutes A handout is shared that lists a series of variables relating to everyday terms. Participants explore how the variables can be combined into expressions, some that have meaning and some that do not. They create a list of meaningful expressions, choose one for a sentence strip and ask the class to guess the meaning of their expression.
- 10–15 minutes Participants reflect on what algebra is and how it is learned. A list of ideas is recorded. The list is then compared to the NCTM Algebra Standard and the NCTM Processing Standards. Participants share ideas on preparing their children for algebra.
- 5 minutes Participants are given a set of tables to complete for homework.

#### Materials

Facilitator	Transparencies (Eng. & Spanish)
<ul style="list-style-type: none"> <li>• Chart paper for Post-its®</li> </ul>	<i>BLM 16: From Tables to Expressions</i> <i>BLM 17.1: Family Expressions</i> <i>BLM 18: NCTM Algebra Standards</i> <i>BLM 19: NCTM Process Standards</i>
Participant	Handouts (English & Spanish)
<ul style="list-style-type: none"> <li>• Pennies and nickels (provided by participants or provide play coins)</li> <li>• Large Post-its®, 1 pad per pair or small group</li> <li>• Markers, 1 per pair or small group</li> <li>• Chart paper, several sheets per pair or group</li> <li>• 3" x 18" Sentence strips, 2 per pair</li> </ul>	<p><b>One set per group for class</b>  <i>BLM 15.1-4: Tables and Expression Cards</i></p> <p><b>One per participant for class</b>  <i>BLM 17.2: Family Expressions</i>  <i>BLM 18: NCTM Algebra Standards</i>  <i>BLM 19: NCTM Process Standards</i></p> <p><b>One per participant for home</b>  <i>BLM 20: At Home with Expressions</i></p>

**Activities**

Preparation of Classroom	Notes
<ol style="list-style-type: none"> <li>1. Post charts from previous sessions to use as needed throughout the session.</li> <li>2. Use <b>Tables and Expression Cards</b> to make sets of cards for each group.</li> <li>3. Set up the <b>Chart It!</b></li> <li>4. Distribute name cards (if applicable).</li> </ol>	
Discussion of Homework (10-25 minutes)	
<ol style="list-style-type: none"> <li>1. As participants arrive have them discuss their discoveries while playing the <b>Money Exchange Game</b> at home.</li> <li>2. After about 5 minutes, lead a whole group discussion on patterns that they have found.</li> <li>3. Have someone use the overhead projector to show how to record the play with two on each side. This might be done with one person making the move while another person records the moves on chart paper. Be sure to ask:             <ul style="list-style-type: none"> <li>• <i>Does anyone have a different way of recording the steps?</i></li> <li>• <i>What kind of a pattern do you see?</i></li> </ul> <p>If no one is able to illustrate how to record moves, walk them through a recording process while someone plays the game on the overhead projector.</p> </li> <li>4. Ask someone to share their recording of the steps for playing with three and four coins on each side. Be sure to ask:             <ul style="list-style-type: none"> <li>• <i>Does anyone have a different way of recording the steps?</i></li> <li>• <i>What kind of a pattern do you see?</i></li> <li>• <i>How can you use this pattern to help you play with more coins?</i></li> </ul> </li> <li>5. Depending on the group, this discussion could lead into an expression that tells how many moves are needed for any number of coins.</li> <li>6. Close this activity by having a brief discussion about the problem solving strategies that they used. Refer to the chart of <b>Problem Solving Strategies</b> which they received during the last session.</li> <li>7. Discuss any questions that participants have about the rest of the homework assignment.</li> </ol>	<p>If the material from the previous session was not completely covered, you may decide to skip playing the game and complete other discussions, presentations, and/or activities.</p> <p>If participants played the game at home and were successful, you may not need to spend much time. This does not usually happen. While few are able to play the game they are not able to record a pattern.</p> <p>Reminder: Notes on playing and discussing the <b>Money Exchange Game</b> are in Session Three.</p> <p>If some finish before others, challenge them to record the steps for five coins on each side.</p> <p>After recording the steps have them play the game by following the steps to see if the pattern works. You can ask,  <i>So, could you play a game with 10 coins one each side?</i></p> <p>Be cautious of the time. It is possible to spend an entire hour on this activity.</p>

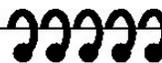
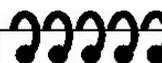
## Activities

Tables and Expression Cards (25 - 35 minutes)	
<p>1. Distribute <b>Tables and Expression Cards, Set A-F</b>, to each group. Display the transparency <b>From Tables to Expressions</b>.</p> <p>2. Have them distribute the cards at their table, each person works individually on a card, writes the missing information on the back of the card and passes it to the person on their right side. The second person to have the card determines the missing information and checks their work with what is written in the back. If there is agreement, the passing of the cards continues. If not, discussion follows often bringing in others and the instructor into the conversation.</p> <p>3. As they are working help those who are stuck by asking some of the following questions:  <b>Set C:</b> <ul style="list-style-type: none"> <li>• <i>What can you do to 2 to get 2.5?</i></li> <li>• <i>Can you do the same thing to 5 and get 5.5?</i></li> <li>• <i>Is 2 increasing or decreasing when it becomes 2.5?</i></li> <li>• <i>What operations might be used to make numbers increase?</i></li> </ul> </p> <p>4. After everyone in the group has completed the cards, have them fill in post-it notes with their expressions. Have them write the letter and the expression on the post-it large enough for everyone to read.</p> <p>5. Post chart paper, each with one of the letters A-F. Have groups put their post-it notes on chart paper.</p> <p>6. Lead a discussion about the answers. To do this read all the answers for A.  <b>Ask:</b> <ul style="list-style-type: none"> <li>• <i>Are all these answers the same?</i></li> <li>• <i>Does everyone agree with all of these?</i></li> <li>• <i>Can anyone prove that this is correct? Incorrect?</i></li> <li>• <i>Does anyone/group want to change their answer?</i></li> </ul> </p> <p>7. <b>Tables and Expressions Cards, Set G-L</b> can be used in one of the following ways: <ul style="list-style-type: none"> <li>a) Give to groups or individuals who are working at a faster pace and need more of a challenge</li> <li>b) Give to everyone as a second activity and facilitate similar to the first.</li> </ul> </p>	<p>The purpose of this activity is to review the use of variables.</p> <p>Requiring participants to work individually ensures that everyone will have time to think on their own. This is important because in some groups, there may be domineering members who do not allow others time to think and/or participate.</p> <p>It is not unusual to have one group working individually and another group working in pairs. If participants work in pairs or small groups, stress that they should first try to think about the problems on their own and then begin a discussion.</p> <p>Use large enough post-its and markers so that the expressions can be seen from afar.</p> <p>It is not necessary to identify the groups who turned in each answer.</p>

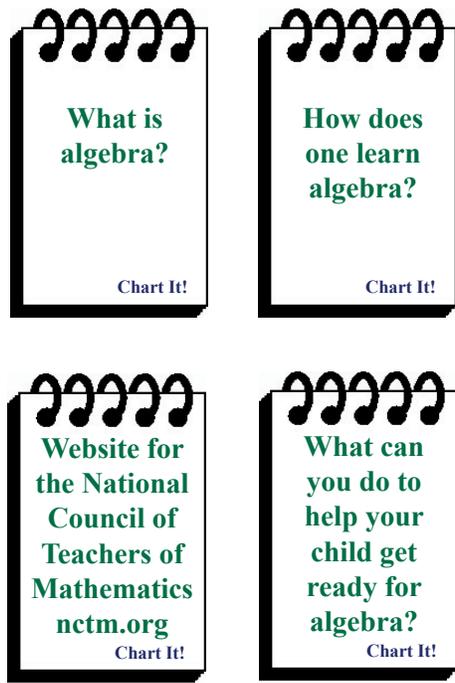
**Activities**

Tables and Expression Cards (continued)	Notes
<p>8. Depending on the time available you may choose to process some, but not all cards from G-L. Having a discussion on Set K will lead to the topic of squaring numbers. Some questions that will help the dialogue are:</p> <ul style="list-style-type: none"> <li>• Does anyone know another way to write <math>b</math> times <math>b</math>?</li> <li>• Does anyone know how to read this (<math>b^2</math>)?</li> </ul> <p>This means <math>b</math> times <math>b</math>. Record <math>b^2</math> on the chart paper. Write the word exponent. Write "b squared" and "b to the 2<sup>nd</sup> power".</p> <p>9. Write some other expressions with exponents and discuss the meaning of them. Example: <math>4^3</math> is read "4 cubed" or "4 to the 3<sup>rd</sup> power". It means to multiply 4 times 4 times 4.</p> <p>10. Let participants know that this is often a concept that will confuse their children. Often students will misinterpret <math>4^3</math> to mean 4 times 3. Tell them mathematics can often be thought of as another language. Learning a language takes practice.</p> <p>11. Distribute one set of blank cards M-X to each group. Ask each person to write an expression on the back of the card. Then use the expression to complete three rows of the table, using either table B or E as the model (one has one ? and the other has 2 ?).</p> <p>12. Have each person trade their card with someone else in their group. Each person then completes the table and finds the expression. They check their work by looking at the expression on the back.</p> <p>13. When a minimum of one round has occurred, ask each group to choose one expression to challenge the class. Give them a sheet of chart paper and markers to record their table.</p> <p>14. Choose presenters by assigning a number to each group and rolling 1-2 dice, depending on the number of groups or using a numbered spinner.</p> <p>15. Make sure that the presenters give the "audience" enough time to try to figure out their expression..</p> <p>16. Continue choosing presenting groups until all groups have presented.</p>	<p>There are usually a few people or groups who need more of a challenge while others are still working on cards A-F. The best way to handle this is to give them cards G-L and let them work ahead. It is okay to not have everyone complete all cards.</p> <p>Be aware that not everyone will understand the exponent notation. Take time to discuss how to read it and write it. You may also want to give a few examples.</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div data-bbox="966 661 1161 976" style="border: 1px solid black; padding: 5px; text-align: center;">  <p><math>b^2</math></p> <p>b times b, exponent b squared b to the 2<sup>nd</sup> power</p> <p>Chart It!</p> </div> <div data-bbox="1209 661 1404 976" style="border: 1px solid black; padding: 5px; text-align: center;">  <p><math>4^3</math></p> <p>4 times 4 times 4; 4 cubed 4 to the 3<sup>rd</sup> power</p> <p>Chart It!</p> </div> </div> <p>There have been times that groups have ended up with complex expressions that are hard to find. As a facilitator you will need to ask the right questions to help the others figure out the expression.</p>

**Activities**

Meaningful Expressions (30 - 40 minutes)	Notes
<p>1. Let the participants know that they will continue their work with expressions.</p> <p>2. Hand out and display <b>Family Expressions</b>. Explain that they will be writing meaningful algebraic expressions with these variables.</p> <p>3. Give examples of expressions that make sense and ones that do not. Record <math>m + w</math> on <b>Chart It!</b>. Ask:</p> <ul style="list-style-type: none"> <li>• <i>What does <math>m</math> stand for? What does <math>w</math> stand for?</i></li> <li>• <i>If we add <math>m</math> and <math>w</math> what does that give us?</i></li> <li>• <i>How could you describe the meaning of <math>m + w</math>?</i></li> </ul> <p>4. After they determine that it means the number of adults in a family, ask:</p> <ul style="list-style-type: none"> <li>• <i>How many men in one family? Women?</i></li> <li>• <i>So, how many adults are in one family?</i></li> </ul> <p>5. Write the expressions <math>g + d</math> and <math>g \times d</math> on <b>Chart It!</b>. Ask:</p> <ul style="list-style-type: none"> <li>• <i>Which makes more sense: <math>g + d</math> or <math>g \times d</math>? Why?</i></li> </ul> <p>6. Ask them to work with a partner to write 4-5 expressions that make sense. Let them know that they can add another symbol if they want.</p> <p>7. Give them 15-20 minutes to work on this. Walk around and answer questions and check for accuracy.</p> <p>8. Distribute 1-2 sentence strips to each partner group. Ask them to write one meaningful expression on each strip. They are not to write the meaning but only the expression.</p> <p>9. Have each group share the expression and ask the others to determine the meaning of the expression.</p> <p>10. As presentations are being made you may want to take note of expressions which you can use to reinforce the concept of an equation.</p> <ol style="list-style-type: none"> <li>a) For example, since <math>m + w + c</math> is the number of people in a family, you may write <math>m + w + c =</math> number of people in a family.</li> <li>b) Ask for a variable that can be used to represent the number of people in a family. The variable, <math>f</math>, is usually given. Write <math>m + w + c = f</math>.</li> </ol>	<div data-bbox="1015 273 1209 577" style="border: 1px solid black; padding: 5px; margin-bottom: 20px;">  <p><b>a meaningful expression:</b> <math>m + w</math></p> <p style="text-align: right; font-size: small;">Chart It!</p> </div> <p>The number of women in a family plus the number of men in a family is the same as the number of adults in a family.</p> <p>It is important to ask these questions to give participants practice understanding the meaning of the variables.</p> <div data-bbox="1015 735 1209 1039" style="border: 1px solid black; padding: 5px; margin-bottom: 20px;">  <p><b>not a meaningful expression:</b> <math>g + d</math></p> <p style="text-align: right; font-size: small;">Chart It!</p> </div> <p>The number of gallons of gas a car uses in a day time plus the cost of one gallon of gasoline does not give a number that means anything.</p> <div data-bbox="1136 1470 1331 1774" style="border: 1px solid black; padding: 5px;">  <p><b>expression:</b> <math>m + w + c</math></p> <p><b>equation:</b> <math>m + w + c = f</math></p> <p style="text-align: right; font-size: small;">Chart It!</p> </div>

**Activities**

Algebra and Learning Algebra (10-15 minutes)	Notes
<p>1. Ask participants to reflect on the following two questions:</p> <ul style="list-style-type: none"> <li>• <i>What is algebra?</i></li> <li>• <i>How does one learn algebra?</i></li> </ul> <p>2. Ask them to write down some ideas on their paper and then discuss them in their groups.</p> <p>3. Ask them to share their ideas with the whole group. Record their ideas on chart paper.</p> <p>4. Display <b>NCTM Algebra Standards</b>. Compare this list to the one that the participants brainstormed. Answer any questions that participants have about the standards.</p> <p>5. Display <b>NCTM Process Standards</b>. Give some brief information on the Professional Standards for School Mathematics. Tell them that the Algebra content standards tell us what mathematics students should be learning and the process standards tell us how students should be learning them. Let them know that these standards are for K-12<sup>th</sup> grade. You may want to give them the web site for NCTM. Additional information on the Process Standards is located under the Facilitator Notes on page 45.</p>	 <p>Website for NCTM Standards:  <a href="http://www.nctm.org/standards/">www.nctm.org/standards/</a></p>
Closure (5 minutes)	
<p>Tell the participants:</p> <p><i>Take a moment to think about how you might help your child get ready for algebra. Then have a few volunteers share their thoughts.</i></p>	
Take Home Activities (5-15 minutes)	
<p>Distribute <b>At Home with Expressions</b> and ask them to complete it for the next session.</p>	
Preparation for the next Session	
<p>1. Collect name cards if applicable.</p> <p>2. Save the <b>Chart It!</b> and bring it to the next session.</p> <p>3. For the next session, you will need CBRs, calculators, and Overhead Viewscreen. These items can be borrowed from the TI loan program. Call 1-800-TI Cares.</p>	

**Facilitator Notes****The Process Standards****Problem Solving Standard**

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

**Reasoning and Proof Standard**

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.

**Communication Standard**

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and other;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

**Connections Standard**

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- recognize and apply mathematics in contexts outside of mathematics.

**Representation Standard**

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

